ATTITUDE CONTROL AND TELESCOPE POINTING FOR WEATHERSAT

CARL A. NARDELL

Michigan Aerospace Corp.

Revisions:

1.0: June 5, 1998, C. Nardell: Initial revision

1.1: June 10, 1998, C. Nardell: Changed inertias to agree with Giacomo Porcelli and found a bug in dynamic model (Izz was being used for Ixz). Changed telescope drive to 2 N·m

2.0: August 19, 1998, C. Nardell: Included more detailed discussion of equations of motion. Includes all terms of the fundamental equation. Updated moments of inertia in simulation results

INTRODUCTION

The WeatherSat spacecraft differs from the traditional view of how spacecraft are designed and built in several ways. For this endeavor to be profitable, it will be necessary to think of the spacecraft and instrument as one entity. In most designs, considerable resources are consumed attempting to identify and reject disturbances introduced to the attitude dynamics of the system from outside influences, as well as disturbances induced from internal systems. Such disturbances can include torques induced by the movement of the instrument if its mass is considerable. Such is the case for WeatherSat. The primary mirror of the telescope is likely to be a massive entity with a high moment of inertia about its axis of rotation. By utilizing a priori knowledge of the motion of this telescope, the control system can be optimized to reject these disturbances. The purpose of this study is twofold: to determine appropriate sizing of momentum wheels as well as a first-order determination of disturbance induced by the telescope; to analyze the merits of feedforward control. An analysis of the dynamics of the system and a scheme for optimizing the control system is presented here. Simulation results are also presented for the dynamic model with and without this control scheme.

PROBLEM FORMULATION

The telescope and the spacecraft can be thought of as two bodies rigidly coupled in two axes, and allowed to spin freely in the other. Figure 1 depicts this simplified view of the system, where the spacecraft is modelled as a cylinder, and the telescope is modelled as a complex body with no assumed symmetry. The two bodies are coupled via a servomotor that is capable of exerting a torque on the bodies about their spin axis. The bodies are assumed to be rigidly coupled in the other two axes with no relative movement.

DYNAMIC SYSTEM

The system has four degrees of freedom. The spacecraft and the telescope rotate together in the roll and pitch axes. The satellite moves in the yaw plane, and the telescope's motion is described as an angle relative to the spacecraft. The notation that will be used to describe these angles is as follows:

ϕ – Roll angle of the satellite and the telescope with respect to the inertial frame of reference

θ – Pitch angle of the satellite and the telescope with respect to the inertial frame of reference

ψ – Yaw angle of the satellite with respect to the inertial frame of reference

α – Angle of telescope relative to satellite

ω_x – Body-centered angular velocity of the spacecraft and telescope about the roll axis

ω_y – Body-centered angular velocity of the spaccraft and telescope about the pitch axis

ω_z – Body-centered angular velocity of the spacecraft about the yaw axis

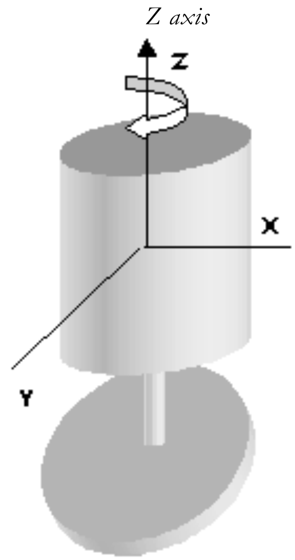

Figure 1: Model of satellite - telescope system

The control inputs to the system consist of four torques. Three of these are torques on the spacecraft exerted by momentum wheels. The fourth is a torque exerted between the spacecraft and the telescope. These will be denoted as M_u, M_v, M_w and M_t respectively.

The equation describing the motion of the telescope is simply

$$M_t = I_t \dot{\alpha} \qquad (1)$$

where I_t is the moment of inertia of the telescope about the axis of rotation.

The equation of motion for the attitude dynamics of the spacecraft is

$$\vec{M} = \frac{d\vec{H}}{dt} + \vec{\omega} \times \mathbf{I}\vec{\omega} \qquad (2)$$

where **I** is the inertia tensor and is of the form

$$\mathbf{I} = \begin{vmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{yx} & I_{yy} & I_{yz} \\ I_{zx} & I_{zy} & I_{zz} \end{vmatrix} \qquad (3)$$

where H is the angular momentum of the system (spacecraft); ω is the inertial angular velocity of the spacecraft; and M is the applied moments from the ACS.

In equation 2, the first term comes from the conservation of momentum. The second term is a kinematic term. Since

$$\vec{H} = \mathbf{I}\vec{\omega}$$

equation 2 can be written as

$$\mathbf{M} = \mathbf{I}\dot{\vec{\omega}} + \dot{\mathbf{I}}\vec{\omega} + \vec{\omega} \times \mathbf{I}\vec{\omega} \qquad (2)$$

For this analysis, it must be assumed that no elements of this matrix are zero. The telescope will contribute to all elements with the exception of I_{zz}. The vector elements of ω are the body-centered angular velocity about each axis, denoted as p, q, and r. The relationship between the body-centered angular velocities and the inertial angular velocities described above can be calculated via a simple coordinate transformation. M consists of the three satellite torque inputs plus the telescope torque:

$$\mathbf{M} = \begin{vmatrix} M_u \\ M_v \\ M_w - M_t \end{vmatrix} \qquad (4)$$

where M_x, M_y, and M_z are moments applied by the momentum wheels, and M_t is the reaction torque to the telescope drive motor.

So as to arrive at the most general solution, no assumption will be made about the symmetry of the spacecraft or telescope. As a result, all terms of the inertia tensor will be non-zero. Because the telescope is attached to the spacecraft via a rigid mount and is stiff in two dimensions, the rotation of the telescope affects four of the values in the inertia tensor. The inertia tensor is calculated to be

$$\mathbf{I} = \begin{vmatrix} I_x^s + \frac{I_x^t + I_y^t}{2} + \frac{I_x^t - I_y^t}{2}\cos 2\alpha - I_{xy}^t \sin 2\alpha & \frac{I_x^t - I_y^t}{2}\sin 2\alpha + I_{xy}^t \cos 2\alpha & I_{xz}^s + I_{xz}^t \\ \frac{I_x^t - I_y^t}{2}\sin 2\alpha + I_{xy}^t \cos 2\alpha & I_y^s + \frac{I_x^t + I_y^t}{2} - \frac{I_x^t - I_y^t}{2}\cos 2\alpha + I_{xy}^t \sin 2\alpha & I_{yz}^s + I_{yz}^t \\ I_{xz}^s + I_{xz}^t & I_{yz}^s + I_{yz}^t & I_z^s \end{vmatrix} \qquad (4)$$

where α is the rotation angle of the telescope; I^s denotes the inertias of the spacecraft and I^t denotes the inertias of the telescope. To simplify the notation, eq. 4 can be written as

$$\mathbf{I} = \begin{vmatrix} I_x(\alpha) & I_{xy}(\alpha) & I_{xz} \\ I_{xy}(\alpha) & I_y(\alpha) & I_{yz} \\ I_{xz} & I_{yz} & I_z \end{vmatrix} \qquad (5)$$

Let's look at each of the three terms in eq. 3 individually. For the kinematic term,

$$\begin{aligned}\vec{\omega}\times\mathbf{I}\vec{\omega} = &\left[I_{xz}\omega_x\omega_y + I_{yz}\left(\omega_y^2-\omega_z^2\right)+\left(I_z - I_y(\alpha)\right)\omega_y\omega_z - I_{xy}(\alpha)\omega_x\omega_z\right]\mathbf{i}+\\ &\left[I_{xy}(\alpha)\omega_y\omega_z + I_{xz}\left(\omega_z^2-\omega_x^2\right)+\left(I_x(\alpha)-I_z\right)\omega_x\omega_z - I_{yz}\omega_x\omega_y\right]\mathbf{j}+\\ &\left[I_{yz}\omega_x\omega_z + I_{xy}(\alpha)\left(\omega_x^2-\omega_y^2\right)+\left(I_y(\alpha)-I_x(\alpha)\right)\omega_x\omega_y - I_{xz}\omega_y\omega_z\right]\mathbf{k}\end{aligned} \qquad (6)$$

The term $\mathbf{I}\frac{d\vec{\omega}}{dt}$ evaluates to

$$\mathbf{I}\frac{d\vec{\omega}}{dt} = \left|\begin{matrix} I_x(\alpha)\frac{d\omega_x}{dt} + I_{xy}(\alpha)\frac{d\omega_y}{dt} + I_z\frac{d\omega_z}{dt} \\ I_{xy}(\alpha)\frac{d\omega_x}{dt} + I_y(\alpha)\frac{d\omega_y}{dt} + I_{yz}\frac{d\omega_z}{dt} \\ I_{xz}\frac{d\omega_x}{dt} + I_{yz}\frac{d\omega_y}{dt} + I_z\frac{d\omega_z}{dt} \end{matrix}\right| \qquad (7)$$

The term $\dot{\mathbf{I}}\vec{\omega}$ comes from the fact that as the telescope rotates, the inertia of the spacecraft is changing. In most satellite systems, this term is small enough that it may be ignored. In this case, the inertia of the telescope is sufficiently large that this term amounts to a significant contribution.

The time derivative of the inertia tensor is

$$\frac{d\mathbf{I}}{dt} = \left|\begin{matrix} \left(\left(I_y^t - I_x^t\right)\sin 2\alpha - 2I_{xy}^t\cos 2\alpha\right)\frac{d\alpha}{dt} & \left(\left(I_x^t - I_y^t\right)\cos 2\alpha - 2I_{xy}^t\sin 2\alpha\right)\frac{d\alpha}{dt} & 0 \\ \left(\left(I_x^t - I_y^t\right)\cos 2\alpha - 2I_{xy}^t\sin 2\alpha\right)\frac{d\alpha}{dt} & \left(\left(I_x^t - I_y^t\right)\sin 2\alpha + 2I_{xy}^t\cos 2\alpha\right)\frac{d\alpha}{dt} & 0 \\ 0 & 0 & 0 \end{matrix}\right| \qquad (8)$$

For notational convenience, eq. 8 can be written as

$$\frac{d\mathbf{I}}{dt} = \left|\begin{matrix} \frac{dI_x}{dt} & \frac{dI_{xy}}{dt} & 0 \\ \frac{dI_{xy}}{dt} & \frac{dI_y}{dt} & 0 \\ 0 & 0 & 0 \end{matrix}\right| \qquad (9)$$

The term $\dot{\mathbf{I}}\vec{\omega}$ then is given by

$$\frac{d\mathbf{I}}{dt}\vec{\varpi} = \begin{vmatrix} \frac{dI_x}{dt}\omega_x + \frac{dI_{xy}}{dt}\omega_y \\ \frac{dI_x}{dt}\omega_x + \frac{dI_{xy}}{dt}\omega_y \\ 0 \end{vmatrix} \qquad (7)$$

The acceleration imparted to each axis can be expressed as

$$\frac{d\vec{\varpi}}{dt} = \left(\vec{M} - \frac{d\mathbf{I}}{dt}\vec{\varpi} - \vec{\varpi}\times\mathbf{I}\vec{\varpi}\right)\mathbf{I}^{-1}$$

These equations can be used in a numerical simulation of the attitude dynamics to solve them for the system response. These equations have been coded in Matlab, and integrated to determine the system response in the presence of rapid telescope articulation.

CONTROL SYSTEM

For the purpose of this analysis, a controller of some type is needed. A simple proportional-derivative controller will be used. A controller of this type is used for each of the four axes. A graphical representation of this controller is shown in figure 2.

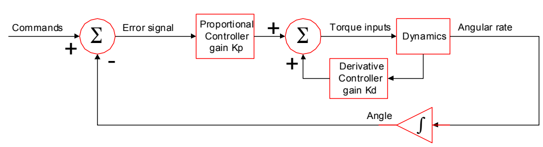

Figure 2: Simple PD controller

For the case of the telescope controller, the control torque, M_t, is described by the control law

$$M_t = k_p\left(\alpha^* - \alpha\right) + k_d\frac{d}{dt}\left(\alpha^* - \alpha\right) \qquad (5)$$

where α^* is the angle being tracked. The gain variables, k_p and k_d, shape the response of the controller, but their values are largely limited by the magnitude of the control torque available. The controllers for the roll, pitch and yaw axes are identical, with the appropriate substitutions for the angle being controlled.

When we introduce the feedforward concept to the control system, the only controller that is affected is the yaw controller. It would take the form

$$M_w = k_p\left(\psi^* - \psi\right) + \frac{d}{dt}\left(\psi^* - \psi\right) + M_t \qquad (6)$$

From a hardware perspective, it may be more convenient to imbed the telescope control input into the error signal. In this case the eq. 6 would take the form

$$M_w = k_p\left(\psi^* - \psi + \frac{M_t}{k_p}\right) + \frac{d}{dt}\left(\psi^* - \psi\right) \quad (7)$$

The control system for the yaw axis is depicted in figure 3.

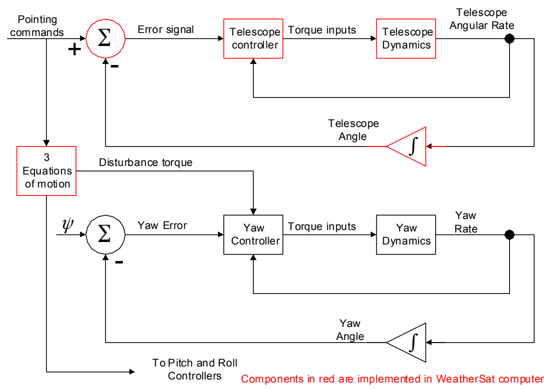

Figure 3: Yaw controller with feedforward term

SIMULATION RESULTS

The telescope on WeatherSat will have a program that cycles continuously. This program consists of a 120 second sequence of 8 45 degree rotations. Nominally, each rotation should have a 3 second rise, with a 12 second dwell. This profile is used as the input for the simulations conducted in this analysis.

The equations of motion in equations 1 and 4 were numerically integrated using a fourth-order Runga-Kutta integration technique. The results were calculated with and without feed-forward knowledge. It should be noted that this simulation does not include effects of aerodynamic torque. Also not included are effects of finite bandwidth on the communication link, finite response of momentum wheels, or friction in the telescope bearing.

Exact moments of inertia are not available at this time, but approximate numbers were used here. The spacecraft was assumed to be a cylinder weighing 800 kg. Values used for the moments of inertia are shown in table 1.

Table 1: Moments of inertia used for simulation

AXIS	VALUE
I_{xx}	706 kg·m²
I_{yy}	316 kg·m²
I_{zz}	444 kg·m²
I_{xy}	3 kg·m²
I_{xz}	3 kg·m²
I_{yz}	3 kg·m²
I_t	3.5 kg·m²

The torque inputs from the momentum wheels are clamped at .3 N ·m for this simulation. A 2. N ·m drive is assumed for the telescope drive.

Figures 4 - 8 show the response of the telescope, roll, pitch and yaw axes without feedforward. Figures 9 - 13 show the same axes with the feedforward term.

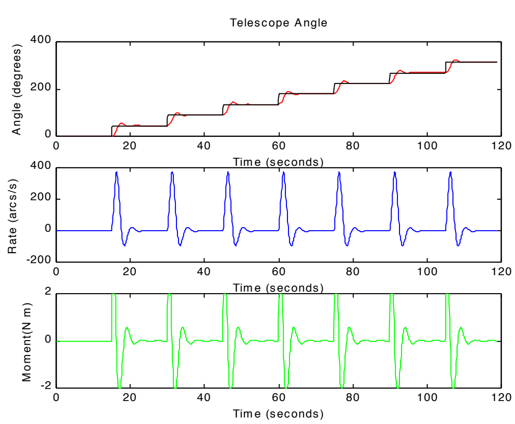

Figure 4: Telescope response without feedforward term

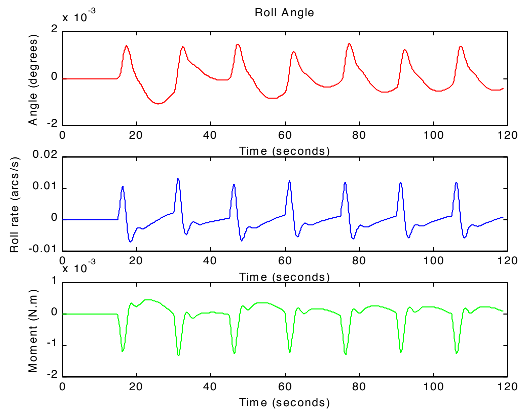

Figure 5: Roll response without feedforward term

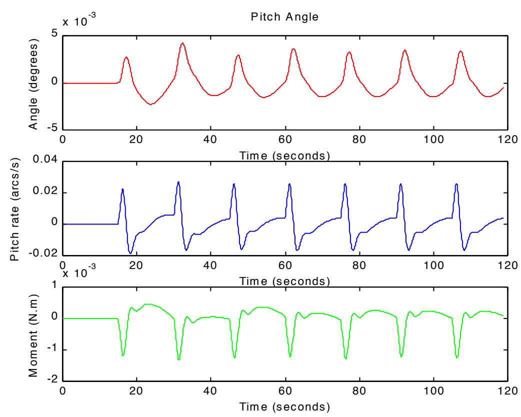

Figure 6: Pitch response without feedforward term

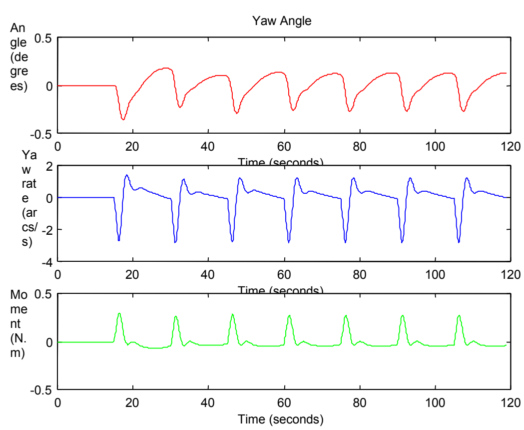

Figure 7: Yaw response without feedforward term

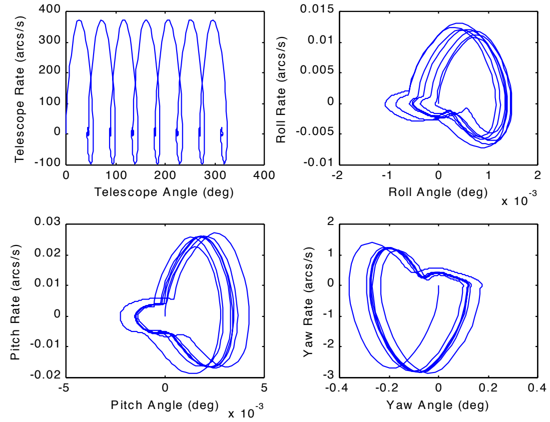

Figure 8: Phase plane plots for all axes without feedforward term

Figure 9: Roll response with feedforward term

Figure 10: Pitch response with feedforward term

Figure 11: Yaw response with feedforward term

CONCLUSIONS

Based on the required pointing accuracy for the telescope of 1 degree, the simulation results presented here indicate that feedforward control is probably not needed for the pointing of the telescope, as coupling between axes does not induce oscillation of sufficient magnitude to impact telescope performance. These results will need to be recalculated when more accurate moments of inertia are available for the spacecraft and telescope. These results are based on fabricated numbers that may or not bear semblance to reality.

These results also indicate that the merits of feedforward are The only cost associated with this increase in control performance is the implementation of a communication link between the instrument computer and the attitude control computer.

It can also be concluded based on these results that .3 N ·m momentum wheels would be appropriate. The exact sizing for the telescope drive will be determined by how tightly the desired profile must be adhered to. The value of 2 N·m will be a good intial estimate.